From Flatness-Based to Model-Free Control Design for Wheeled Vehicles

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Outline

Outline

Vehicle dynamics behavior

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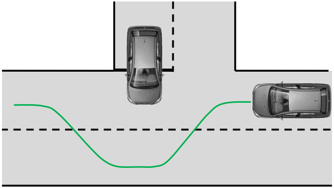

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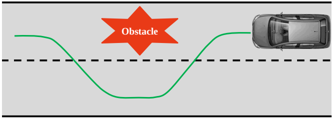


Combined longitudinal and lateral control to perform some coupled maneuvers

Coupled maneuvers

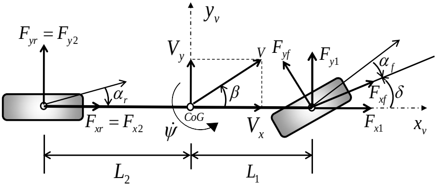
- Lane-change maneuvers
- Obstacle avoidance
- Combined lane-keeping and steering control during critical driving situations
- ...

Coupled nonlinear 3DoF Two wheels vehicle model

$$\left\{\begin{array}{l} ma_x = m(\dot{V}_x - \dot{\psi} V_y) = (F_{x1} + F_{x2}) \\ ma_y = m(\dot{V}_y + \dot{\psi} V_x) = (F_{y1} + F_{y2}) \\ I_z \ddot{\psi} = M_{z1} + M_{z2} \end{array}\right\}$$

$$\Leftrightarrow \left\{\begin{array}{c} \dot{V}_x \\ \dot{V}_y \\ \ddot{\psi} \end{array}\right\} = \dot{x} = f(x,t) + g(x,t)u + g_1 u_1 u_2 + g_2 u_2^2$$

- Inputs: $u_1 = C_\omega$ acceleration/braking torque and $u_2 = \delta$ steering wheel angle
- F_{xi} and F_{yi} depend nonlinearly on u_1 and u_2.



Coupled nonlinear 3DoF Two wheels vehicle model

$$\dot{x} = f(x,t) + g(x,t)u + g_1 u_1 u_2 + g_2 u_2^2$$

where:

$$f(x,t) = \begin{bmatrix} \dot{\psi} V_y - \frac{I_\omega}{mR}(\dot{\omega}_r + \dot{\omega}_f) \\ -\dot{\psi} V_x + \frac{1}{m}\left(-C_f\left(\frac{V_y + L_1\dot{\psi}}{V_x}\right) - C_r\left(\frac{V_y - L_2\dot{\psi}}{V_x}\right)\right) \\ \frac{1}{I_z}\left(-L_1 C_f\left(\frac{V_y + L_1\dot{\psi}}{V_x}\right) + L_2 C_r\left(\frac{V_y - L_2\dot{\psi}}{V_x}\right)\right) \end{bmatrix},$$

$$g(x,t) = \begin{bmatrix} \frac{1}{mR} & \frac{C_f}{m}\left(\frac{V_y + L_1\dot{\psi}}{V_x}\right) \\ 0 & (C_f R - I_\omega \dot{\omega}_f)/mR \\ 0 & (L_1 C_f R - L_1 I_\omega \dot{\omega}_f)/I_z R \end{bmatrix}, g_1 = \begin{bmatrix} 0 \\ \frac{1}{mR} \\ \frac{L_1}{I_z R} \end{bmatrix}, g_2 = \begin{bmatrix} \frac{-C_f}{m} \\ 0 \\ 0 \end{bmatrix}.$$

Remark: nonlinear terms such as $u_1 u_2$ and u_2^2 are neglected, then the following system is considered:

$$\dot{x} = f(x,t) + g(x,t)u$$

Coupled nonlinear 3DoF Two wheels vehicle model

Coupled nonlinear vehicle model for control design

$$\left\{ \begin{array}{c} \dot{V}_x \\ \dot{V}_y \\ \ddot{\psi} \end{array} \right\} = f(x,t) + g(x,t)u$$

- Two inputs
  1. Acceleration/Braking Torque

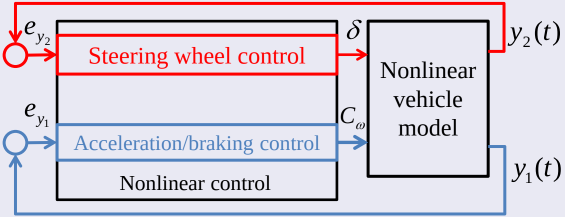
$$u_1 = C_\omega \Leftrightarrow \left\{ \begin{array}{lll} u_1 = T_b & \textbf{if} & \text{Braking Torque} \\ u_1 = T_m & \textbf{if} & \text{Acceleration Torque} \end{array} \right\}$$

  2. Steering wheel angle $u_2 = \delta$.
- Two outputs:
  1. Longitudinal motion.
  2. Combined lateral and yaw motions.

Longitudinal and lateral control

General block diagram of control approach

- Control inputs — Steering wheel angle $\delta(t)$ and acceleration/braking torque C_ω.
- Outputs — longitudinal speed $V_x(t)$ and yaw rate $\dot{\psi}(t)$.



Control and estimation approaches

- Flatness control.
- Model-free control.
- Algebraic identification techniques.

Outline

Flatness based nonlinear longitudinal/lateral control

Flatness property: Definition

Consider the system

$$\dot{x} = F(x, u)$$

- there exists a vector-valued function h such that
$$y = h(x, u, \dot{u}, \cdots, u^{(r)})$$
where $y = (y, \cdots, y_m) \in \mathbb{R}^m$, $r \in \mathbb{N}$;
- the components of $x = (x, \cdots, x_n)$ and $u = (u, \cdots, u_m)$ may be expressed as
$$x = A(y, \dot{y}, \cdots, y^{(r_x)}), \quad r_x \in \mathbb{N}$$
$$u = B(y, \dot{y}, \cdots, y^{(r_u)}), \quad r_u \in \mathbb{N}$$

Remember that y in $\dot{x} = F(x, u)$ is called a *flat output*.

Flatness based nonlinear longitudinal/lateral control

Flatness proof of the nonlinear two wheels vehicle model (1/2)

We want to show that y_1 and y_2 given by

- First output: the longitudinal speed $y_1 = V_x$
- Second output: the angular momentum of a point on the axis between the centers of the front and rear axles $y_2 = L_f m V_y - I_z \dot{\psi}$

define flat outputs.
Some algebraic manipulations yield

$$x = \begin{bmatrix} V_x \\ V_y \\ \dot{\psi} \end{bmatrix} = A(y_1, y_2, \dot{y}_2) = \begin{bmatrix} y_1 \\ \frac{y_2}{L_f m} - \left(\frac{I_z}{L_f m}\right)\left(\frac{L_f m y_1 \dot{y}_2 + C_r(L_f+L_r)y_2}{C_r(L_f+L_r)(I_z - L_r L_f m) + (L_f m y_1)^2}\right) \\ -\left(\frac{L_f m y_1 \dot{y}_2 + C_r(L_f+L_r)y_2}{C_r(L_f+L_r)(I_z - L_r L_f m) + (L_f m y_1)^2}\right) \end{bmatrix}$$

and $$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = B(y, \dot{y}, \cdots, y^{(r_u)}) = \Delta^{-1}(y_1, y_2, \dot{y}_2)\left(\begin{bmatrix} \dot{y}_1 \\ \ddot{y}_2 \end{bmatrix} - \Phi(y_1, y_2, \dot{y}_2)\right)$$

More details are given in [L. Menhour, B. d'Andréa-Novel, M. Fliess and H. Mounier, "Coupled nonlinear vehicle control: Flatness-based setting with algebraic estimation techniques", *Control Engin. Practice*, vol. 22, 135-146, 2014.]

Flatness based nonlinear longitudinal/lateral control

Flatness proof of the nonlinear two wheels vehicle model (2/2)

The flatness property holds therefore if the matrix Δ is invertible.

$$\det(\Delta(y_1, y_2, \dot{y}_2)) = \frac{(I_\omega \dot{\omega}_f - C_f R)\left(L_f^2 y_1^2 m^2 - C_r(L_f + L_r)L_r L_f m + C_r I_z L\right)}{I_z R^2 y_1 m^2} \neq 0$$

- $\dot{\omega}_f << RC_f/I_\omega$, then: RC_f/I_ω is around 10^4, then $I_\omega \dot{\omega}_f - C_f R \neq 0$.
- $I_z > L_f m$, then: $\left(L_f^2 y_1^2 m^2 - C_r(L_f + L_r)L_r L_f m + C_r I_z L\right) \neq 0$.

Therefore y_1 and y_2 constitute flat outputs for our system. Then, we have

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \Delta^{-1}(y_1, y_2, \dot{y}_2)\left(\begin{bmatrix} \dot{y}_1 \\ \ddot{y}_2 \end{bmatrix} - \Phi(y_1, y_2, \dot{y}_2)\right)$$

Δ being invertible, we can obtain the following closed-loop controller:

$$\begin{bmatrix} \dot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} \dot{y}_1^{ref} + K_1^1 e_{y_1} + K_1^2 \int e_{y_1} dt \\ \ddot{y}_2^{ref} + K_2^1 \dot{e}_{y_2} + K_2^2 e_{y_2} + K_2^3 \int e_{y_2} dt \end{bmatrix}$$

where, $e_{y_1} = y_1^{ref} - y_1 = V_x^{ref} - V_x$ and $e_{y_2} = y_2^{ref} - y_2$.

Flatness based nonlinear longitudinal/lateral control

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where, $e_{y_1} = y_1^{ref} - y_1 = V_x^{ref} - V_x$ and $e_{y_2} = y_2^{ref} - y_2$.

Flatness-based control and algebraic estimation

Flatness-based control law and algebraic estimation

$$\delta(t) = f_\delta(y_1^{ref}, y_2^{ref}, \dot{y}_1^{ref}, \dot{y}_2^{ref}, \ddot{y}_2^{ref}, y_1, y_2, \dot{y}_2)$$

$$C_\omega(t) = f_{C_\omega}(y_1^{ref}, y_2^{ref}, \dot{y}_1^{ref}, \dot{y}_2^{ref}, \ddot{y}_2^{ref}, y_1, y_2, \dot{y}_2)$$

Algebraic estimation methods will be be used, as detailed later on, to filter the signals involved and their time derivatives.
[Fliess and Sira-Ramírez 2008]

Model-Free longitudinal and lateral control: Motivation

Difficulty to obtain a realistic vehicle model

It is very hard or sometimes impossible to obtain a realistic vehicle model due to parametric uncertainties and unknown dynamics

- Tire cornering stiffness coefficients (C_f and C_r).
- Aerodynamic forces.
- Road geometry: road bank and slop angles.
- Road adherence.
- Vehicle mass.
- Position of center of gravity.
- ...

Model-Free longitudinal and lateral control: Motivation

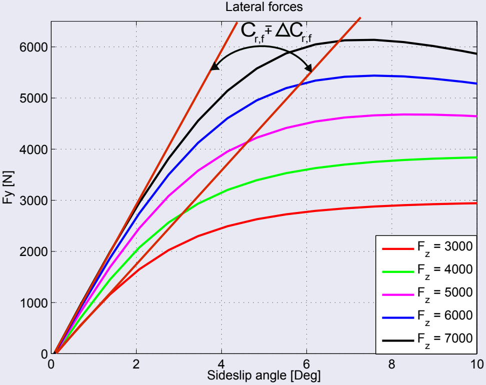
Parameter variations

Example of linear tire models

- Linear tire models:

$$\begin{cases} F_{yf} = C_f \left(\delta - \beta - \frac{L_f \dot{\psi}}{V_x} \right) \\ F_{yr} = -C_r \left(\beta - \frac{L_r \dot{\psi}}{V_x} \right) \end{cases}$$

- Variation of sideslip angle and vertical forces F_{zi}



Theoretical tire characteristics

Model-Free longitudinal and lateral control: Motivation

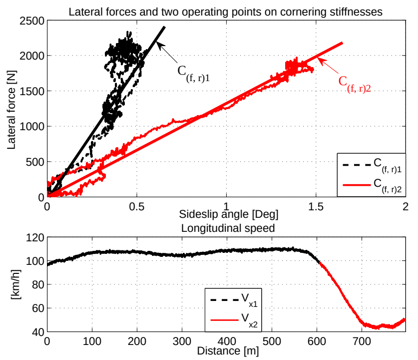
Parameter variations

Example of linear tire models

- Linear tire models:

$$\begin{cases} F_{yf} = C_f \left(\delta - \beta - \frac{L_f \dot{\psi}}{V_x}\right) \\ F_{yr} = -C_r \left(\beta - \frac{L_r \dot{\psi}}{V_x}\right) \end{cases}$$

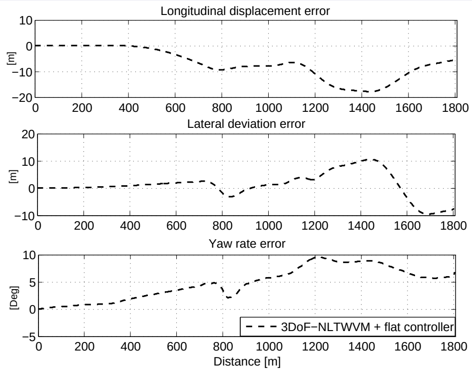
- Variation of longitudinal speed V_x



Two slopes of cornering stiffnesses during an experimental braking maneuver

Model-Free longitudinal and lateral control: Motivation

Previous results: Nominal vehicle flatness-based control



Simulation test with $C_f * 0.3$ and $C_r * 0.3$: trajectory tracking with nonlinear flatness-based vehicle control

More details are given in [L. Menhour, B. d'Andréa-Novel, M. Fliess, H. Mounier, "Multivariable decoupled longitudinal and lateral vehicle control: A model-free design", IEEE Conf. Decision Control, Florence, 2013.]

Model-Free Control: Background

Local model

The following local model can be used for Model-Free control approach

$$y^{(\nu)} = F + \alpha u$$

- ν is the derivation order, which is equal, in most of the cases, to 1 or 2.
- α is a constant parameter and can be chosen by the user.

In a lot of cases ν is chosen equal to 1, the local model then becomes

$$\dot{y} = F + \alpha u$$

and the desired behavior is obtained using the following i-PI controller:

$$\boxed{u = \frac{\left(-F + \dot{y}^d + K_p e + K_i \int e dt\right)}{\alpha}}$$

with $e = y - y^d$ is the tracking error, y^d is the desired output trajectory, K_p, K_i, K_d are gains of the i-PI.

Algebraic estimation

Basic principles

Aim: find an estimate of the first derivative of a signal

$$y(t) = a_0 + a_1 t, \quad a_0, \; a_1 \in \mathbb{R}$$

$$\overbrace{\left\{\begin{array}{lcl} Y(s) & = & \frac{a_0}{s} + \frac{a_1}{s^2} \\ sY(s) & = & a_0 + \frac{a_1}{s} \\ \frac{d}{ds}\left(sY(s)\right) & = & -\frac{a_1}{s^2} \quad \Leftrightarrow \quad \underbrace{Y(s) + s\frac{dY(s)}{ds} = -\frac{a_1}{s^2}} \end{array}\right.}^{\textbf{Use powers of s and derivatives w.r.t. s}}$$

$$\Downarrow$$

Use negative powers of s to filter the signal

$$\Downarrow$$

$$\frac{Y(s)}{s^2} + \frac{1}{s}\frac{dY(s)}{ds} = -\frac{a_1}{s^4}$$

Algebraic estimation

Basic principles

$$\frac{Y(s)}{s^2} + \frac{1}{s}\frac{dY(s)}{ds} = -\frac{a_1}{s^4}$$

$$\Downarrow \quad \boxed{\begin{cases} \frac{c}{s^\alpha}, \quad \alpha \geq 1, \quad c \in \mathbb{C} \Rightarrow c\frac{t^{\alpha-1}}{(\alpha-1)}, \quad t > 0 \\ \frac{1}{s^\alpha}\frac{d^n y}{ds^n}, \quad \Rightarrow \int_0^t \int_0^{t_{\alpha-1}} \cdots \int_0^{t_1} (-1)^n \tau^n y(\tau) dt_1 \cdots dt_{\alpha-1} dt \\ \qquad \Rightarrow \frac{(-1)^n}{(\alpha-1)!} \int_0^t (t-\tau)^{\alpha-1} \tau^n y(\tau) d\tau \end{cases}}$$

$$\hat{a}_1 = -\frac{3!}{T^3} \int_0^T (T - 2\tau) y(\tau) d\tau$$

Algebraic estimation

Basic principles

$$d_m(t) = d_0 + d_1 t, \quad t \leq 0, \quad d_0, \quad d_1 \in \mathbb{R}$$

$$\boxed{d_0 = \frac{2}{T^2} \int_0^T (2T - 3\tau) d_m(\tau) d\tau}$$

$$\boxed{d_1 = -\frac{6}{T^3} \int_0^T (T - 2\tau) d_m(\tau) d\tau}$$

- $d_0(t)$ is a natural filter of the measured signal $d_m(t)$.
- We have studied the properties of these numerical filters.

  [F. Garcia Collado, B. d'A-N, M. Fliess, H. Mounier, GRETSI, 2009]

Algebraic estimation of F

Estimation of F

The following local model

$$\dot{y} = F + \alpha u$$

can be rewritten in the operational domain as follows:

$$sY = \frac{\Phi}{s} + \alpha U + y(0)$$

After some manipulations in the operational domain, it yields in the time domain the following real time estimator of F

$$\boxed{F_{\text{est}}(t) = -\frac{6}{\tau^3}\int_{t-\tau}^{t} \left[(\tau - 2\sigma)y(\sigma) + \alpha\sigma(\tau - \sigma)u(\sigma)\right] d\sigma}$$

where τ is a small time window.

For more details see [M. Fliess and C. Join, "Model-free control", Int. J. Control, 86(12), pages 2228-2252, 2013]

Decoupled Vehicle control based on Model-Free control

Control design scheme

The design of longitudinal and lateral vehicle control requires:

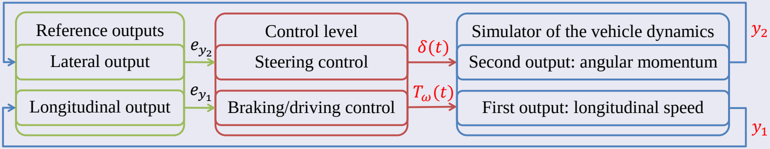
- Two Control Inputs
  - Control of longitudinal motion via acceleration/braking torque: $u_1 = C_\omega$.
  - Control of lateral motion via steering angle: $u_2 = \delta$.
- Two outputs

Model Free longitudinal and lateral vehicle control with the flat outputs

Model Free Control with the flat outputs

The flat outputs are considered in the model free control context

$$\begin{cases} y_1 = V_x \\ y_2 = L_f m V_y - I_z \dot{\psi} \end{cases}$$



Model Free longitudinal and lateral vehicle control with the flat outputs

Model Free Control with the flat outputs

Based on the flatness property, the following two sub-models are naturally considered:

- Longitudinal local model:

$$y_1^{(\nu_1)} = F_1 + \alpha_1 u_1$$

- Lateral local model:

$$y_2^{(\nu_2)} = F_2 + \alpha_2 u_2$$

with $\nu_1 = 1$ and $\nu_2 = 2$.

More details are given in [L. Menhour, B. d'Andréa-Novel, M. Fliess, H. Mounier, "Multivariable decoupled longitudinal and lateral vehicle control: A model-free design", IEEE Conf. Decision Control, Florence, 2013.]

Decoupled Vehicle control based on Model-Free control with natural outputs

Model-Free control with natural outputs

It must be pointed out that the two flat outputs depend on geometrical and inertial parameters. So, to be totally parameter-independent, an idea in the context of MFC is to consider the following natural outputs, which can be obtained through direct measurements:

$$\begin{cases} y_1 = \text{Longitudinal speed} \\ y_2 = \text{Lateral deviation} \end{cases}$$

Based on the above natural vehicle outputs, the following two sub-models are considered:

- Longitudinal local model:

$$y_1^{(\nu_1)} = F_1 + \alpha_1 u_1$$

- Lateral local model:

$$y_2^{(\nu_2)} = F_2 + \alpha_2 u_2$$

with $\nu_1 = 1$ and $\nu_2 = 2$.

Model-free control with natural outputs

Model-free control and algebraic estimation

- Longitudinal i-P controller:

$$u_1 = \frac{1}{\alpha_1}\left(-\hat{F}_1 + \hat{\dot{y}}_1^d - K_p^{y_1} e_{y_1}\right)$$

and the estimated value of $\hat{F}_1$ is given by the following algebraic estimator:

$$\hat{F}_1(t) = -\frac{6}{\tau^3}\int_{t-\tau}^{t}(\tau - 2\sigma)y_1(\sigma)d\sigma - \frac{6\alpha}{\tau^3}\int_{t-\tau}^{t}\sigma(\tau - 2\sigma)u_1(\sigma)d\sigma$$

- Lateral i-PD controller:

$$u_2 = \frac{1}{\alpha_2}\left(-\hat{F}_2 + \hat{\ddot{y}}_2^d - K_d^{y_2}\dot{e}_{y_2} - K_p^{y_2} e_{y_2}\right)$$

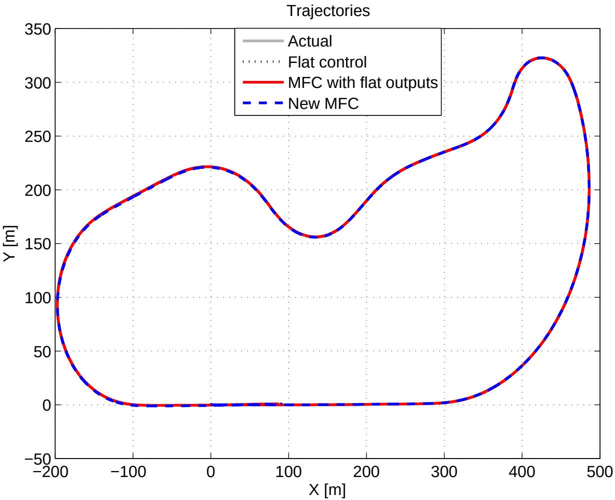
and $\hat{F}_2$ is given also by the following algebraic estimator:

$$\hat{F}_2(t) = -\frac{60}{\tau^5}\int_{t-\tau}^{t}(\tau^2 + 6\sigma^2 - 6\tau\sigma)y_2(\sigma)d\sigma - \frac{30\alpha}{\tau^5}\int_{t-\tau}^{t}(\tau - \sigma)^2\sigma^2 u_2(\sigma)d\sigma$$

Outline

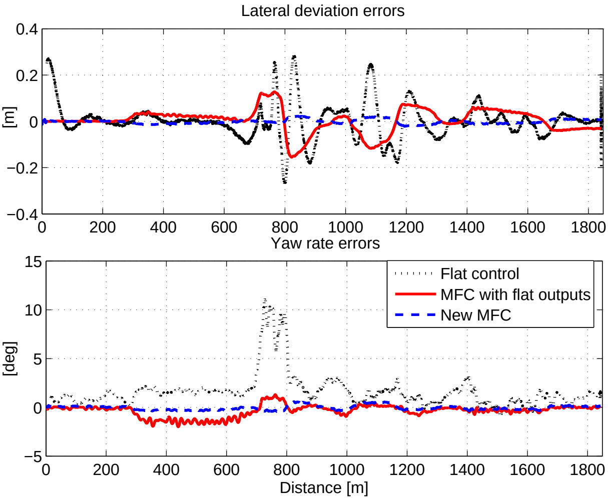
1. Problem statement
2. Controllers design
   - Flatness-based control
   - Model-Free Control
3. Validation results
4. Conclusions

Controllers validation



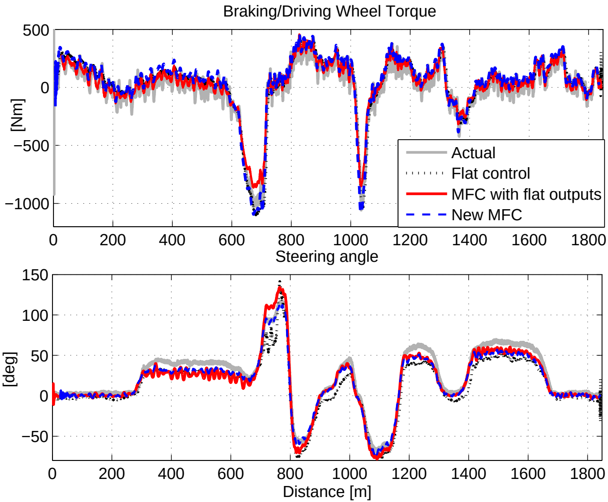
Desired trajectory versus the closed-loop trajectories with the different controllers

Controllers validation



Tracking trajectory errors on lateral deviation and yaw angle

Controllers validation



Torques and steering wheel angle control signals: actual and those obtained with the three controllers

Outline

Conclusion

- This new MFC strategy has been shown to be efficient in the context of global chassis control, using firstly a Matlab simulation environment but also using a more realistic interconnected SiVIC/RTMaps platform.
- It is naturally robust w.r.t. parameter uncertainties and the choice of the i-P or i-PD gains is quite easy.

Thank you very much
Questions?

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